Determine whether each expression below is valid. If it is valid, state whether the result is a scalar or a vector. **SCORE: ____ / 10 PTS**

[a] $(\vec{u} \cdot \vec{v})(\vec{u} \times \vec{v})$ VALID, VECTOR

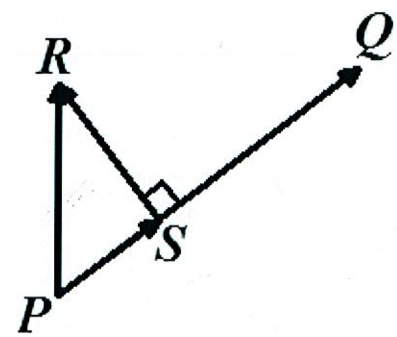
[b] $\vec{u} \times (\vec{v} \cdot \vec{w})$ INVALID

2½ EACH

[c] $(\vec{u} \times \vec{v}) \cdot (\vec{w} \cdot \vec{s})$ INVALID

Let P be the point $(-1, -5, 1)$, R be the point $(0, -7, -2)$, and $\overrightarrow{PQ}$ be the vector $3\vec{i} + \vec{j} - 2\vec{k}$ in the diagram on the right. **NOTE: ∠RSQ is a right angle.** **SCORE: ____ / 106 PTS**

Diagram not drawn to scale



NOTE: The diagram is NOT drawn to scale.

[a] Write "I UNDERSTAND THAT THE DIAGRAM IS NOT DRAWN TO SCALE" to indicate that you understand that the diagram is **NOT** drawn to scale.

[b] Find symmetric equations for the line passing through P and R.

4

$\overrightarrow{PR} = \langle 0 - -1, -7 - -5, -2 - 1 \rangle = \langle 1, -2, -3 \rangle$

6 $\quad x + 1 = -\frac{y+5}{2} = -\frac{z-1}{3}$ OR $x = -\frac{y+7}{2} = -\frac{z+2}{3}$

[c] Find $\angle QPR$. **NOTE: The diagram is NOT drawn to scale.**

$$\cos^{-1} \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \cos^{-1} \frac{\overbrace{3 - 2 + 6}^{4}}{\underbrace{\sqrt{14}}_{4} \underbrace{\sqrt{14}}_{4}} = \cos^{-1} \frac{\cancel{7}^{1}}{\cancel{14}_{2}} = \underbrace{60^\circ \text{ or } \frac{\pi}{3}}_{4}$$

[d] Find $\overrightarrow{SR}$.

$$\overrightarrow{PS} = \underbrace{\text{PROJ}_{\overrightarrow{PQ}} \overrightarrow{PR}}_{5} = \frac{\overrightarrow{PR} \cdot \overrightarrow{PQ}}{\overrightarrow{PQ} \cdot \overrightarrow{PQ}} \overrightarrow{PQ} = \frac{\overbrace{\cancel{7}^{1}}^{4}}{\underbrace{\cancel{14}_{2}}_{4}} \langle 3, 1, -2 \rangle = \underbrace{\langle \tfrac{3}{2}, \tfrac{1}{2}, -1 \rangle}_{4}$$

$$\overrightarrow{SR} = \overrightarrow{PR} - \overrightarrow{PS} = \langle 1, -2, -3 \rangle - \langle \tfrac{3}{2}, \tfrac{1}{2}, -1 \rangle = \underbrace{\langle -\tfrac{1}{2}, -\tfrac{5}{2}, -2 \rangle}_{4}$$

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⬅ CONTINUED FROM PREVIOUS PAGE

[e] Find symmetric equations of the line passing through P that is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{PQ}$.

$\vec{d} = \overrightarrow{PR} \times \overrightarrow{PQ} = \langle 1,-2,-3\rangle \times \langle 3,1,-2\rangle = \langle 4--3, -9--2, 1--6\rangle$

$= \langle 7,-7,7\rangle$ 9

USE $\langle 1,-1,1\rangle$ INSTEAD SINCE PARALLEL + SIMPLER

6 $x+1 = -(y+5) = z-1$

SANITY CHECK: $1+2-3=0$ ✓ $3-1-2=0$ ✓ 4

[f] Find parametric equations of the line passing through R that is perpendicular to the plane $3y-2z+7=0$.

$\vec{d} = \langle 0,3,-2\rangle$ 4

2 2 2

$x = 0$

$y = -7+3t$

$z = -2-2t$

[g] Find the general form ($Ax+By+Cz+D=0$) of the equation of the plane passing through P, Q and R.

$\vec{n} = \overrightarrow{PR} \times \overrightarrow{PQ} = \langle 7,-7,7\rangle$ 4 USE $\langle 1,-1,1\rangle$ INSTEAD

6 $1(x-0) - 1(y+7) + 1(z+2) = 0$

$x - y + z - 5 = 0$ 4

[h] If T (not shown) is a point such that $PQTR$ is a parallelogram, find the area of $PQTR$.

$\|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \sqrt{7^2+(-7)^2+7^2} = 7\sqrt{3}$

4 4

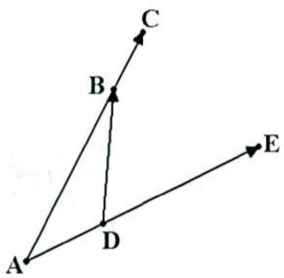
[i] If M (not shown) is a point such that $\overrightarrow{PM}$ is a unit vector and the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PM}$ is $\frac{5\pi}{6}$, find $\|\overrightarrow{PQ} \times \overrightarrow{PM}\|$.

$\|\overrightarrow{PQ}\|\,\|\overrightarrow{PM}\| \sin\frac{5\pi}{6} = \sqrt{14}(1)(\frac{1}{2}) = \frac{1}{2}\sqrt{14}$

4 4

In the diagram below, AE is three times the length of AD, and AC is four times the length of BC. **SCORE: ____ / 10 PTS**

If $\vec{u} = \overrightarrow{AE}$ and $\vec{w} = \overrightarrow{AC}$, find an expression for $\overrightarrow{DB}$ in terms of $\vec{u}$ and $\vec{w}$.



$\overrightarrow{BC} = \frac{1}{4}\overrightarrow{AC} \longrightarrow \overrightarrow{AB} = \frac{3}{4}\overrightarrow{AC} = \frac{3}{4}\vec{w}$

2 ORDER OF SUBTRACTION

$\overrightarrow{AD} = \frac{1}{3}\overrightarrow{AE} = \frac{1}{3}\vec{u}$

3

$\overrightarrow{DB} = \overrightarrow{AB} - \overrightarrow{AD} = \frac{3}{4}\vec{w} - \frac{1}{3}\vec{u}$

3 2

FILL IN THE BLANKS. **SCORE: ____ / 12 PTS**

[a] If a plane and a line are perpendicular, then the NORMAL (1) vector of the plane is PARALLEL (2) to the DIRECTION (1) vector of the line.

[b] If plane 1 and plane 2 are parallel, then the NORMAL vector of plane 1 is PARALLEL to the NORMAL vector of plane 2. **<u>NOTE: ALL 3 BLANKS MUST BE CORRECT TO EARN ANY CREDIT.</u>**

2

[c] The equation of the xz – plane is $y = 0$. 3

[d] If $\vec{u} \cdot \vec{u} = 8$, then $\|\vec{u}\| = 2\sqrt{2}$. 3

Let $\vec{p}$ be the vector $< -4\sqrt{3}, -4 >$, and $\vec{q}$ be the vector with magnitude 12 and direction angle $\frac{5\pi}{3}$. **SCORE: ____ / 12 PTS**

[a] Find the direction angle of $\vec{p}$.

$$\pi + \tan^{-1}\frac{-4}{-4\sqrt{3}} = \pi + \tan^{-1}\frac{\sqrt{3}}{3}$$

$$= \pi + \frac{\pi}{6}$$

2 EACH

$$= \frac{7\pi}{6} \text{ or } 210^\circ$$

[b] Write $\vec{q}$ as a linear combination of $\vec{i}$ and $\vec{j}$.

$$\langle 12\cos\tfrac{5\pi}{3}, 12\sin\tfrac{5\pi}{3}\rangle = \langle 12(\tfrac{1}{2}), 12(-\tfrac{\sqrt{3}}{2})\rangle$$

$$= 6\vec{i} - 6\sqrt{3}\vec{j}$$